ECC in FLOSS

More curves to the set

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Collaborora

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FOSDEM

Presentation Outline

ECC overview

ECC standards

Implementations

Newer curves, more standards

Choose your curve

COLLABORATION

Elliptic curves

What's an elliptic curve?

COLLABORA

Elliptic curves

What's an elliptic curve?

Elliptic curve

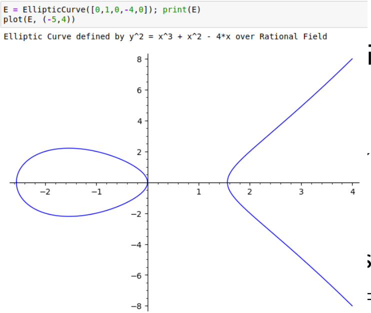
An elliptic curve over an odd field is a modular congruency with this odd number:

$$y^2 \equiv x^3 + ax + b \, (mod \, p)$$

where $a, b \in \mathbb{F}_p$
With the condition that it must **not** have singular points (aka non zero discriminant $\Delta = 4a^3 + 27b^2 \neq 0$)

Elliptic curves

What's an elliptic curve?



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$+ b \, (mod \; p)$

st **not** have singular points

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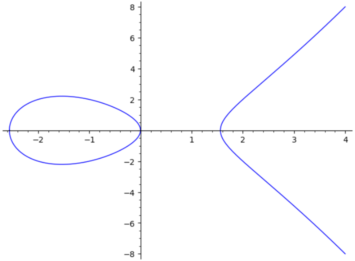
Elliptic curves

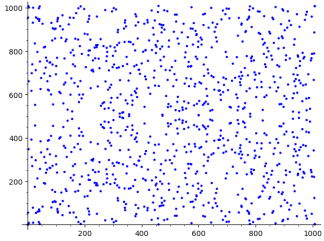
```
E = EllipticCurve([0,1,0,-4,0]); print(E)
plot(E, (-5,4))
```

```
Elliptic Curve defined by y^2 = x^3 + x^2 - 4*x over Rational Field
```




```
E_gf = E.change_ring(GF(1009)); print(E_gf)
plot(E_gf)
```

```
Elliptic Curve defined by y^2 = x^3 + x^2 + 1005*x over Finite Field of size 1009
```



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Points on this elliptic curve

The set of points of the elliptic curve is $E/\mathbb{F}_p \cup \mathcal{O}_E$.

$$E(\mathbb{F}_p) =$$

$$\{(x, y) \in \mathbb{F}_p^2 : y^2 = x^3 + ax + b\,(mod\; p) | \Delta \neq 0\}$$

$$\cup$$

$$\{\mathcal{O}_E\}$$

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Cyclic group for crypto

A cyclic subgroup of points over an elliptic curve:

$$\langle G \rangle = \{G, 2G, 3G, \ldots, \mathcal{O}_E = nG\}$$

where n is the order of the cyclic group. We need a big n, and $n \approx \#E(\mathbb{F}_p)$, because $|\langle G \rangle| \approx |E(\mathbb{F}_p)|$

$$h = \frac{\#E/\mathbb{F}_p}{n}$$

* On Discrete Logarithm Problem over finite fields you'll see notation like $y = g^x$ when in ec you see $Q = dP$

** Skip torsion points or Isogenies definitions

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What's an elliptic curve?

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$$y^2 = x^3 + ax + b \text{ over } \mathbb{F}_p$$
$$y^2 + xy = x^3 + ax^2 + b \text{ over } \mathbb{F}_{2^m}$$

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$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$$

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  This has a huge effect on cryptanalysis
  and the lifespan on embedded

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- NIST 186-2
- ANSI X9.62-1998[a]
- Certicom Sec1v1 & Sec2v1

[a] Annex H.2: share curve means share cryptoanalysis

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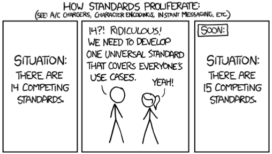

xkcd 927

[a] Annex H.2: share curve means share cryptoanalysis

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Presentation Outline

Implementations

	WRF	Edwards
OpenSSL	✓✓	✓
libgcrypt (GnuPG)	✓	✓
GnuTLS	✓	✓
Kernel	✓	✓
WolfSSL	✓	✓
crypto (rust)		✓
sequoia (rust)	✓	✓
cryptography (python)		
elliptic-py	✓	
elliptic (javascript)	✓	✓
crypto (go)	✓	✓

$*$ Not pretending to be exhaustive

✓✓ $E(\mathbb{F}_p)$ and $E(\mathbb{F}_{2^m})$

Implementations

- tor
  - torspec: rend-spec-v3[a]
    - `onion_address = base32(PUBKEY | CHECKSUM | VERSION) + ".onion`

[a] v2: was a 80-bit truncated SHA1 of a 1024 RSA key, onion addresses were 16 characters long

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    - `PUBKEY`: is the 32 bytes ed25519 master pubkey of the hidden service
    - The result is a 56-characters onion address
  - The key must not have torsion component
    (or multiple equivalent onion addresses could map to the same service).
    This is related with the `cofactor`.

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They are Montgomery curves with a birationally equivalent [twisted] Edwards maps.

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- Curve25519 & Curve448
  - rfc7748: Few primes of the form $2^c - s$ with $<<$ s exist in $[2^{250}, 2^{521}]$

$$y^2 = x^3 + Ax^2 + x$$

p	$2^{255} - 19$	$2^{448} - 2^{224} - 1$
A	486662	156326
h	8	4

NIST

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$$x^2 + y^2 = a + dx^2y^2$$

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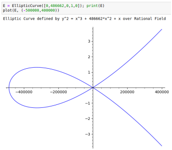

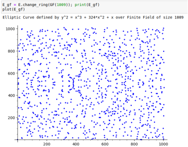

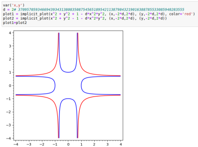

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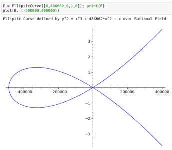

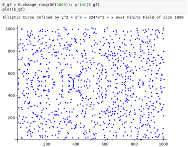

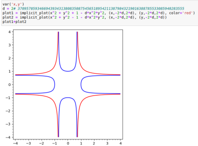

- Why they are good?
  - Build to avoid potential implementation pitfaills,
  - Immune to timing attacks,

Double-odd [Jacobi Quartic]

- `do255{e,s}`
  - curve $y^2 = x(x^2 + ax + b)$ order $2r \equiv 2 \ (mod\,4)$
  - Different base field and curves by operation:

encryption	$p = 2^{255} - 18651$	$(a, b) = (0, -2)$
sign	$p = 2^{255} - 3957$	$(a, b) = (-1, \frac{1}{2})$
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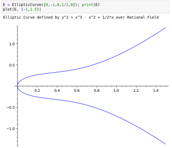

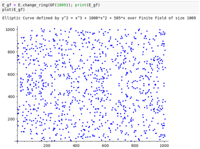

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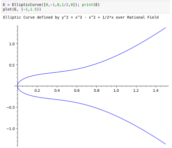
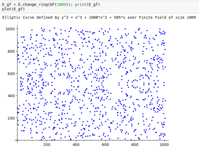
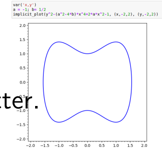




- `jq255{e,s}`
  - Another mapping to a Jacobi Quartic: $y^2 = (a^2 - 4b)x^4 - 2ax^2 + 1$
  - Coordinates transformations and operations in the maps here they are better.
  - Even faster operations and shorter signatures



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    - Everyone can **influence and verify**
    - But no one can knowingly affect the choices
    - The results cannot be predicted or effectively manipulated
    - **Prevent prior cryptanalysis** or target malicious choices.

ristretto255, decaf448 and the zoo

Corollary

A random zoo: sloth, unicorn, and trx

"Is a way to **fix the small set of elliptic curves** *currently used, and it allows usage of parameters that are frequently refreshed and that cannot have been scrutinised before"*

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ECC in FLOSS

Thanks!

Q & A