UNIVERSITÄT ZU LÜBECK
INSTITUTE FOR
THEORETICAL COMPUTER SCIENCE

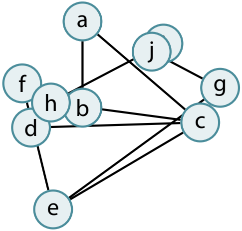
Algorithmic Graph Drawing in Ti*k*Z with Lua

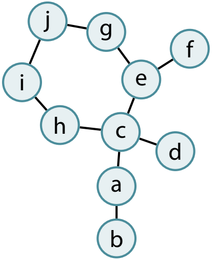
Till Tantau

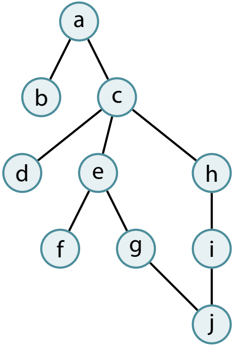
FOSDEM 2015

IM FOCUS DAS LEBEN

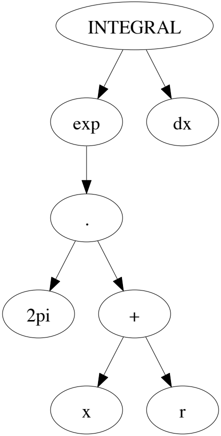
Which drawing of the graph would you choose?

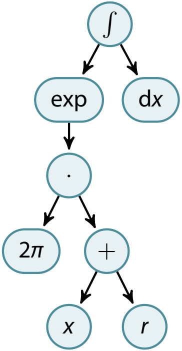






Typography and graph drawing do not mix easily.





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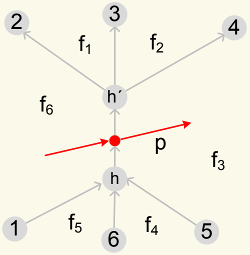
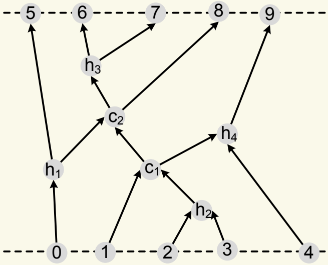


(b) A realization of p.

al arc can reduce the crossings

aints, are restricted to the



(a)

3. Steps towards a final layout: (a) PR $\mathcal{R}$, (b) fine-layering of the subgra

Outline

Graph Drawing

- Aims
- Solutions: Force-Based Methods
- Solutions: The Sugiyama Method

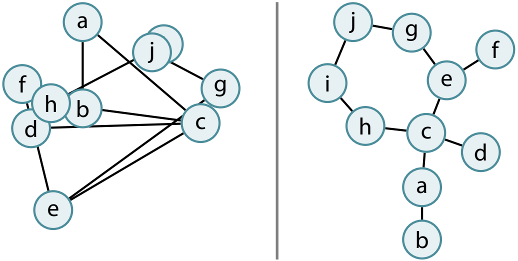
Graph Drawing in Ti*k*Z

- What is Ti*k*Z?
- How to Draw a Graph with Ti*k*Z

Graph Drawing in Ti*k*Z with Lua

- Programming in TeX
- Programming in Lua
- How to Implement a Graph Drawing Algorithm

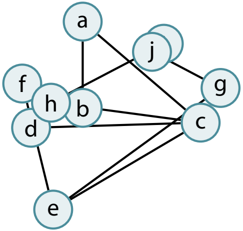
Why is the right drawing better than the left one?

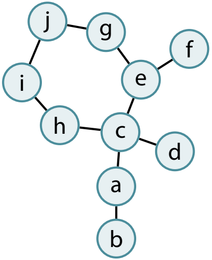


Some observations:

1. On the right, there are less *crossings*.
2. On the right, there are less *overlaps*.
3. On the right, there are more *symmetries*.
4. On the right, the *edge lengths* are similar.
5. On the right, the *angles between edges* are similar.

Why is the right drawing better than the left one?





This leads to an *optimzation problem:* "Draw the graph such that

1. *edge crossings* are minimized,
2. *node overlaps* are minimized,
3. *symmetries* are maximized,
4. *deviations in edge lengths* are minimized
5. *angular variance* is minimized."

There are many other possible objectives: Important stuff near the center, clustered clusters, . . .

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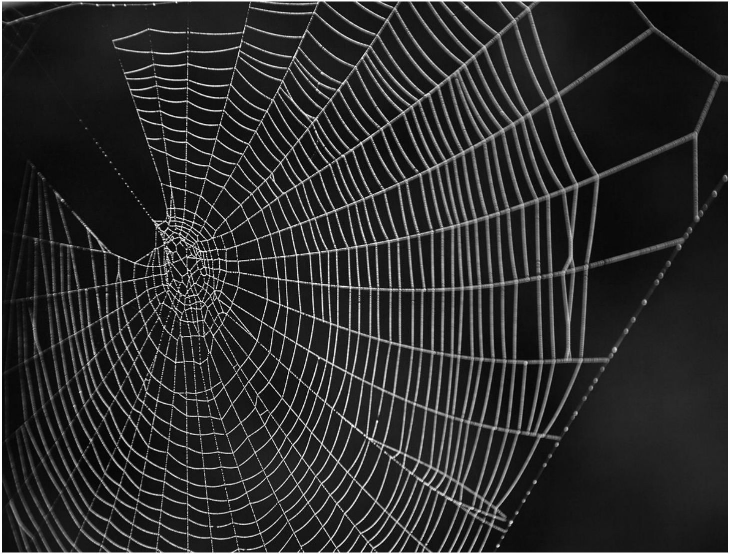
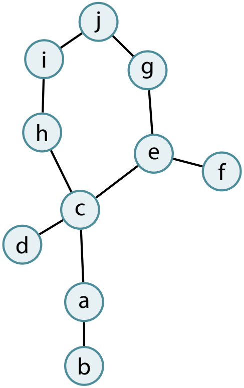
Mother nature draws graphs beautifully.



The force-based approach: Apply forces to nodes iteratively.

- Nodes are *movable*.
- Edges cause *forces* between nodes.
- We *simulate* the resulting node movements until an *equilibrium* is reached.



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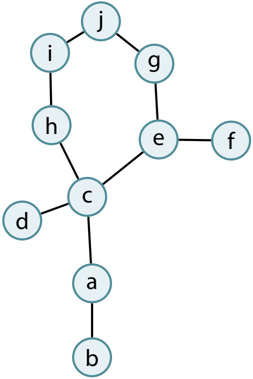

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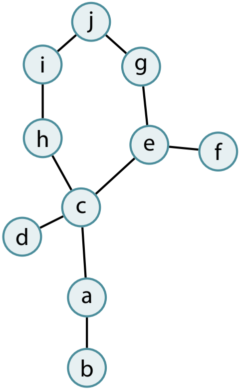

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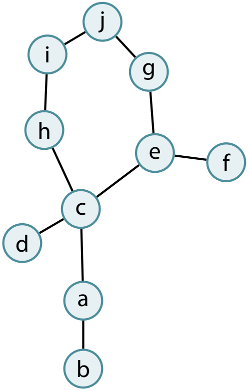

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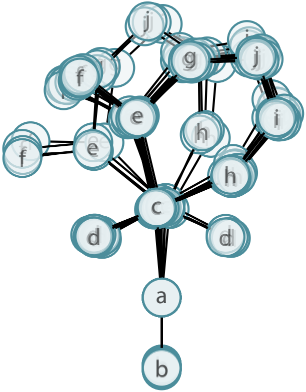

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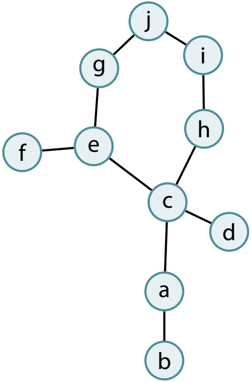

Forces 1 (Tutte): Spring forces

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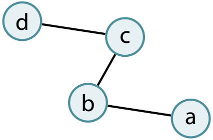
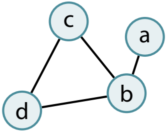
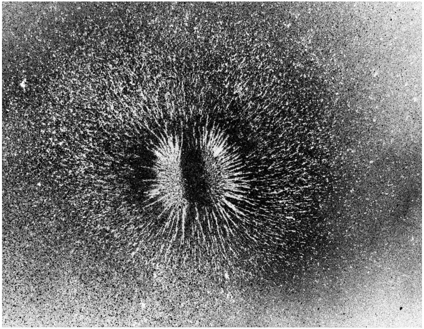
- Edges are *springs*.
- Springs have a *natural length*.
- If an edge is too short, "it pushes the nodes apart."
- If an edge is too long, "it pulls the nodes together."

Forces 1 (Tutte): Spring forces



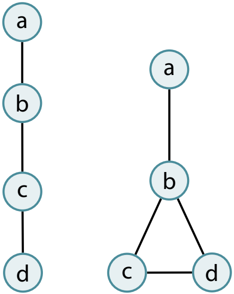


Forces 2 (Eades): Electrical forces

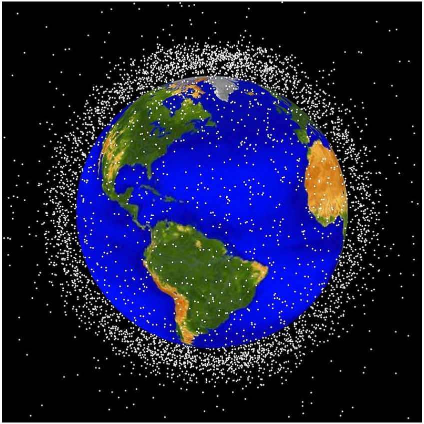


- There are additional *repulsive forces* between nodes.
- Nodes hence tend to form *circles* and *lines*.
- Angles tend to be equal.

Forces 2 (Eades): Electrical forces



Forces 3: Gravitational forces



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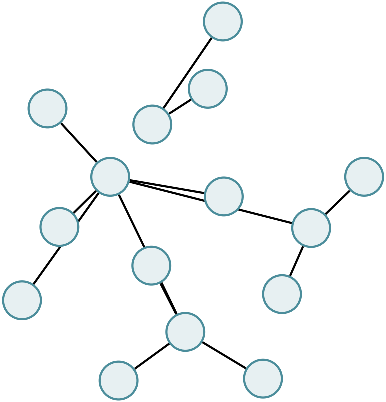
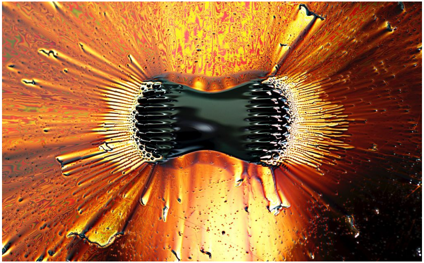
- Nodes are additionally pulled to the center.
- The "heavy, important" nodes tend to be in the center.

Forces 3: Gravitational forces



Forces 4: Magnetic fields



GNU Free Documentation License, Author Gregory F. Maxwell

- Edges try to align with the direction of a force field.
- For instance, we can cause edges to become *horizontal* or *vertical*.

Summary of force-based algorithms.

Advantages

+ "There is a force for every aesthetic objective."
+ Easy iterative implementation.
+ Edge routing is easily incorporated.

Disadvantages

- Iterative algorithms are slow.
- Difficult to implement *well*.
- Difficult to reproduce and predict drawings.

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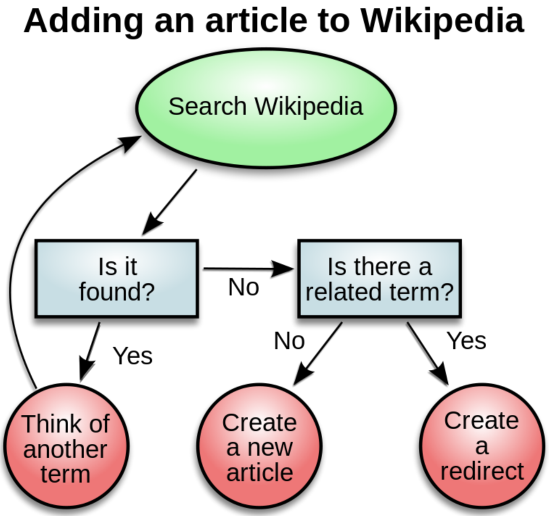
Many graphs are layered.



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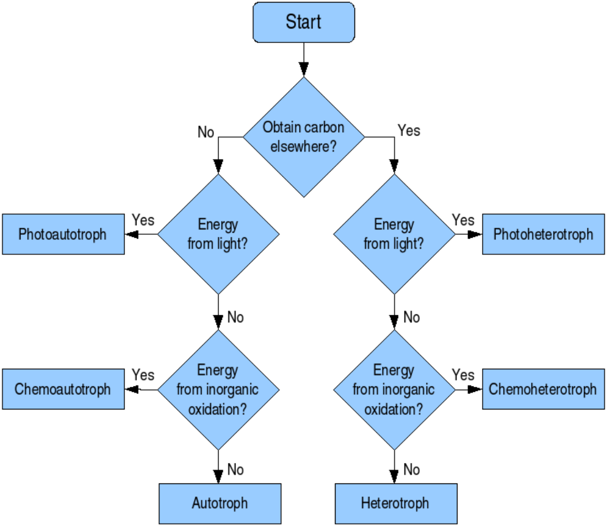
Many graphs are layered.



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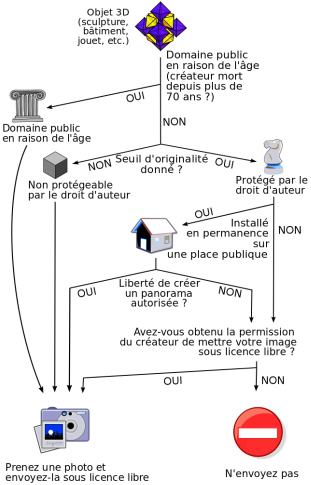

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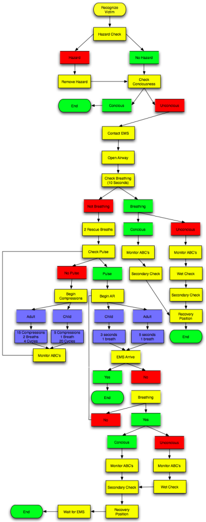

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The Sugiyama method.

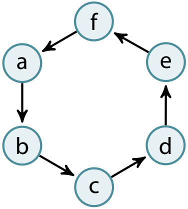
On input of a *directed graph*, do:

1. Make the graph *acyclic*, if necessary.
2. Assign a *layer* to each node, so that edges are only between nodes of adjacent layers.
3. Minimize the number of *edge crossings.*
4. Position the nodes on each layer *nicely.*

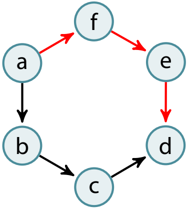
(Unfortunately, all but the last step are NP-complete. . .)

Sugiyama step 1: Make the graph acyclic

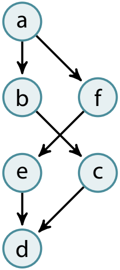
Input



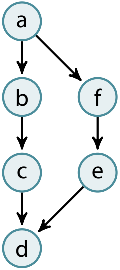
Redirecting edges makes it acyclic



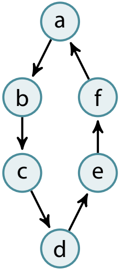
Sugiyama step 2: Assign layers



Sugiyama step 3: Minimize crossings



Sugiyama step 4: Position nodes nicely



Summary of Sugiyama's method

Advantages

+ Produces nice drawings of layered graphs.
+ Can handle edges crossing several layers.
+ Extremely fast when good heuristics are used.

Disadvantages

– Difficult implementation.
– Works only for inherently layered graphs.

Outline

TikZ ist *kein* Zeichenprogramm

```
Let $\int_0^1 \sqrt{x}\, dx$
be the integral\dots
```

Let $\int_0^1 \sqrt{x}\, dx$ be the integral. . .

```
Let \tikz \fill[red]
  (0,0) circle[radius=1mm];
be the circle\dots
```

Let ● be the circle. . .

- TikZ is a library of *TeX macros* for specifying graphics.
- I developed it about 10 years ago in order to produce the 10 figures of my PhD thesis.
- Today, the manual has over a 1000 pages.

How does it work?

```
The triangle \tikz \draw (0,0)
  -- (30:10pt) -- (60:10pt) -- cycle;
```

The triangle 

Ti*k*Z first transforms the code into a *series of graphics commands*:

```
\pgfpathmoveto{\pgfpointxy{0}{0}}
\pgfpathlineto{\pgfpointpolar{30}{10pt}}
\pgfpathlineto{\pgfpointpolar{60}{10pt}}
\pgfpathclose
\pgfusepath{draw}
```

How does it work?

```
The triangle \tikz \draw (0,0)
  -- (30:10pt) -- (60:10pt) -- cycle;
```

The triangle 

These, in turn, get transformed into *abstract graphics primitives*:

```
\pgfsys@moveto{0pt}{0pt}
\pgfsys@lineto{8.660254pt}{5pt}
\pgfsys@lineto{5pt}{8.660254pt}
\pgfsys@closepath
\pgfsys@stroke
```

How does it work?

```
The triangle \tikz \draw (0,0)
  -- (30:10pt) -- (60:10pt) -- cycle;
```

The triangle 

Finally, these are translated into *concrete graphics primitives:*

```
\special{ps:: 0 0 moveto}
\special{ps:: 8.627899 4.98132 lineto}
\special{ps:: 4.98132 8.627899 lineto}
\special{ps:: closepath}
\special{ps:: stroke}
```

(for PostScript output)

How does it work?

```
The triangle \tikz \draw (0,0)
  -- (30:10pt) -- (60:10pt) -- cycle;
```

The triangle 

Finally, these are translated into *concrete graphics primitives:*

```
\special{pdf: 0 0 m}
\special{pdf: 8.627899 4.98132 l}
\special{pdf: 4.98132 8.627899 l}
\special{pdf: h}
\special{pdf: S}
```

(for PDF output)

How does it work?

```
The triangle \tikz \draw (0,0)
  -- (30:10pt) -- (60:10pt) -- cycle;
```

The triangle 

Finally, these are translated into *concrete graphics primitives:*

```
\special{dvisvgm:raw
  <path d = " M 0 0
              L 8.660254 5
              L 5 8.660254
              Z"
        style = "stroke">}
```

(for SVG output)

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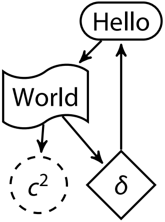
The TikZ syntax for graphs

- A succinct, well-designed syntax for graphs is important when *humans* specify graphs *“by hand”.*
- The TikZ syntax *mixes the philosophies* of DOT and TikZ.

```
\tikz \graph {
      Hello    [rounded rectangle]
  ->  World    [tape]
  ->  "$c^2$"  [circle, dashed];

  World
  ->  "$\delta$" [diamond]
  ->  Hello;
};
```



The TikZ syntax for graphs

- *Node options follow nodes.*
- Edge options follow edges.
- Special notation for edges.
- Natural syntax for trees.

```
\tikz \graph {
     Hello    [rounded rectangle]
 ->  World    [tape]
 ->  "$c^2$"  [circle, dashed];

 World
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```

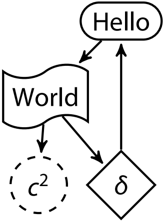

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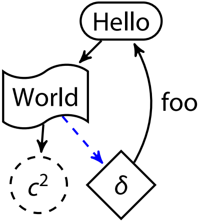
```
\tikz \graph {
     Hello    [rounded rectangle]
  -> World    [tape]
  -> "$c^2$"  [circle, dashed];

  World
  ->[dashed, blue] "$\delta$"[diamond]
  ->[bend right, "foo"'] Hello;
};
```



The TikZ syntax for graphs

- Node options follow nodes.
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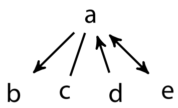
```
\tikz \graph {
  a -> b -- c <- d <-> e;
};
```



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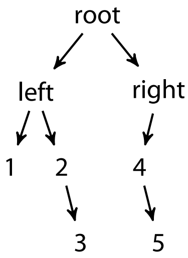
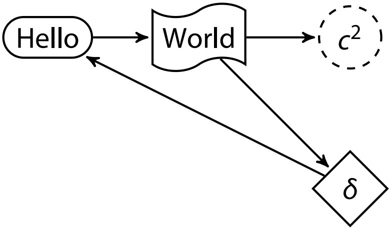
- Special notation for edges.
- *Natural syntax for trees.*

```
\tikz \graph [binary tree layout] {
  root -> {
    left -> {
      1,
      2 -> 3 [second]
    },
    right -> {
      4 -> { , 5 }
    }
  }
};
```



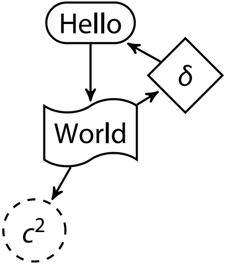
Explicit coordinates . . .

```
\tikz \graph {
  Hello        [x=0, y=2, rounded rectangle];
  World        [x=2, y=2, tape];
  "$c^2$"      [x=4, y=2, circle, dashed];
  "$\delta$"   [x=4, y=0, diamond];

  Hello -> World -> "$c^2$";
  World -> "$\delta$" -> Hello;
};
```



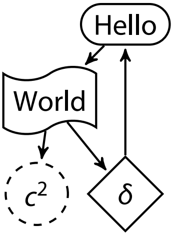
. . . versus algorithmic graph drawing.

```
\usegdlibrary{force}
\tikz \graph [spring layout, node distance=1.2cm] {
  Hello      [x=0, y=2, rounded rectangle];
  World      [x=2, y=2, tape];
  "$c^2$"    [x=4, y=2, circle, dashed];
  "$\delta$" [x=4, y=0, diamond];

  Hello -> World -> "$c^2$";
  World -> "$\delta$" -> Hello;
};
```

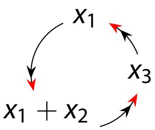


. . . versus algorithmic graph drawing.

```
\usegdlibrary{layered}
\tikz \graph [layered layout] {
 Hello      [           rounded rectangle];
 World      [           tape];
 "$c^2$"    [           circle, dashed];
 "$\delta$" [           diamond];

 Hello -> World -> "$c^2$";
 World -> "$\delta$" -> Hello;
};
```

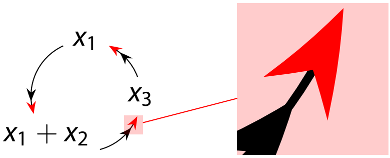


Graph drawing is even useful in seemingly trivial cases.




```
\tikz [>={Stealth[bend] Stealth[bend, red]}]
  \graph [simple necklace layout, necklace routing] {
    "$x_1$" -> "$x_1+x_2$" -> "$x_3$" -> "$x_1$"
  };
```

Graph drawing is even useful in seemingly trivial cases.




```
\tikz [>={Stealth[bend] Stealth[bend, red]}]
  \graph [simple necklace layout, necklace routing] {
   "$x_1$" -> "$x_1+x_2$" -> "$x_3$" -> "$x_1$"
  };
```

Outline

TEX und LuaTEX

TEX is great, but . . .

- implementing large algorithms is *practically impossible* since
- we miss *floating point numbers*, *strings*, *control structures*, *arrays*, *records*, *modules*, . . .

```
The sum of the first 100 numbers is
\newcount\i \newcount\sum
\loop
  \advance\sum by \i\relax
  \ifnum\i<100
  \advance\i by 1\relax
\repeat
\the\sum
```

The sum of the first 100 numbers is 5050

TEX und LuaTEX

TEX is great, but . . .

- implementing large algorithms is *practically impossible* since
- we miss *floating point numbers*, *strings*, *control structures*, *arrays*, *records*, *modules*, . . .

Lua is a minimalistic, elegant language, . . .

- . . . that has been integrated into recent TEX versions:

```
The sum of the first 100 numbers is
\directlua{
  local sum = 0
  for i=1,100 do
    sum = sum + i
  end
  tex.print(sum)
}
```

The sum of the first 100 numbers is 5050

Lua by examples: Hello World.

```
print "Hello World!"
```

- Lua is an imperative scripting language. . .
- . . . that gets you going quickly . . .
- . . . and is really tiny (compiler and libraries around 200kB).

Lua by examples: Variables and types.

```
local x = 1
local y = 2
local z = "Hello there"

if 2*x == y then
  print "Ok"
end

if z == "Hello there" then
  print "Ok"
end
```

- The syntax is a bit “Pascal-like”.
- There are only few types (numbers, strings, functions, tables).
- You cannot and need not specify types.

Lua by examples: Functions.

```
function factorial (n)
  if n <= 1 then
    return 1
  else
    return n*factorial(n-1)
  end
end
```

- Functions are first-order citizens.
- They can be passed around and closures are fully supported.

Lua by examples: Everything is a (hash) table.

```
local array1 = { 2, 3, "hallo" }
local array2 = { 4, 3, 2, 1 }
local record = {
  start = 1,
  stop = 2
}
```

Lua's "everything is a table" paradigm:

- An *array* hashes positive integers to entries.
- A *struct* hashes strings to entries
  (so `record.start` is syntactic sugar for `record["start"]`).

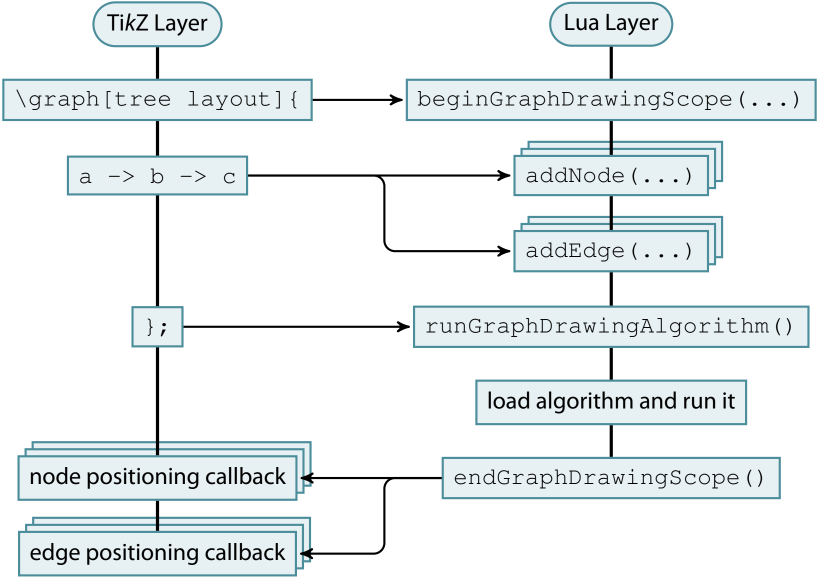
Lua's hash tables are

- *incredibly fast* (strings are prehashed, integers are not hashed but internally form an array),
- *incredibly easy* (they grow and shrink automatically, the syntax is very well designed).

Lua: What else?

- Lua supports coroutines.
- Lua supports meta-programming and thereby classes and objects.
- Lua does not crash.
- Lua does garbage collection.
- Lua integrates seamlessly with C in both directions.

The interplay of Lua and TikZ.



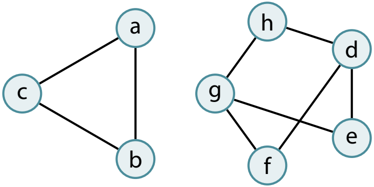
Outline

Complete code for a very simple graph drawing algorithm.

```lua
-- File SimpleDemo.lua
local MyAlgorithmClass = {}
function MyAlgorithmClass:run()
 local g = self.digraph
 local alpha = (2 * math.pi) / #g.vertices
 local radius = g.options.radius
 for i,vertex in ipairs(g.vertices) do
  vertex.pos.x = radius * math.cos(i * alpha)
  vertex.pos.y = radius * math.sin(i * alpha)
 end
end

-- "Publish" the algorithm
local graph_drawing_framework =
 require "pgf.gd.interface.InterfaceToAlgorithms"
graph_drawing_framework.declare {
 key           = "simple demo layout",
 algorithm     = MyAlgorithmClass,
 preconditions = { connected = true }
}
```

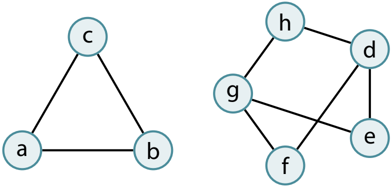
We can immediately use the algorithm – no compilation or installation is needed.

```
\usegdlibrary{SimpleDemo}
...
\tikz \graph [ simple demo layout, radius=1cm ] {
 a -- b -- c -- a;
 d -- e;
 f -- g -- h -- d -- f;
 e -- g;
};
```



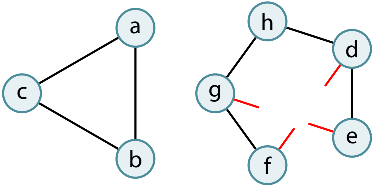
We can immediately use the algorithm – no compilation or installation is needed.

```
\usegdlibrary{SimpleDemo}
...
\tikz \graph [ simple demo layout, radius=1cm] {
  a --[orient=right] b -- c -- a;
  d -- e;
  f -- g -- h -- d -- f;
  e -- g;
};
```



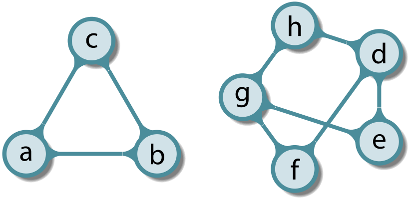
We can immediately use the algorithm – no compilation or installation is needed.

```
\usegdlibrary{SimpleDemo}
...
\tikz \graph [ simple demo layout, radius=1cm]
 a --[orient=right] b -- c -- a;
 d -- e;
 f -- g -- h -- d --[stub,red] f;
 e --[stub, red] g;
};
```



We can immediately use the algorithm – no compilation or installation is needed.

```
\usegdlibrary{SimpleDemo}
...
\tikz \graph [ simple demo layout, radius=1cm,
               nodes={circle, fill=..., ...},
               edges={circle connection bar, ...}] {
  a --[orient=right] b -- c -- a;
  d -- e;
  f -- g -- h -- d -- f;
  e -- g;
};
```



Summary

Graph drawing is about drawing graphs

- quickly,
- such that some aesthetic criteria are met and
- such that structure in the graphs becomes visible.

Graph drawing in TikZ is directed at

- *users* who wish to draw graphs as part of TEX documents
- and *researchers* who implement new algorithms.

Graph drawing in TikZwith Lua means that

- algorithms can and must be implement in the Lua language
- inside a *framework* that takes care of common tasks.